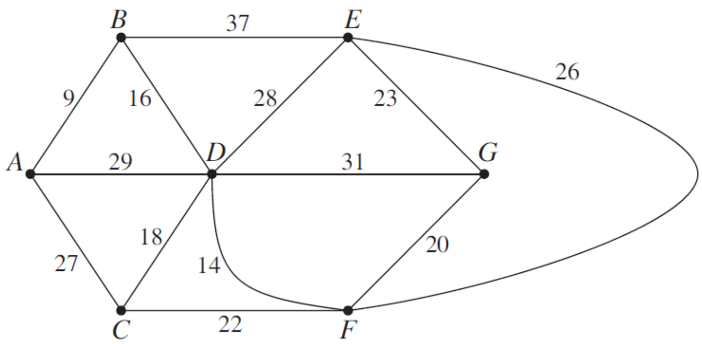
Decision Mathematics 1
Trees, Prim's and Kruskal's Algorithms

Key Definitions:

1.) Tree.
2.) 'Minimum Spanning Tree' or 'Minimum Connector'.

Examples:

1.) January 2009 – Question 3, parts i and ii on Kruskal's Algorithm:



(i) This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. **[5]**

(ii) Use your answer to part **(i)** to find the weight of a minimum spanning tree for the network with vertex E, and all the arcs joined to E, removed. Hence find a lower bound for the travelling salesperson problem on the original network. **[3]**

2.) June 2007 – Question 6 part i – Matrix Version of Prim's Algorithm:

The table shows the distances, in miles, along the direct roads between six villages, A to F. A dash (–) indicates that there is no direct road linking the villages.

	A	B	C	D	E	F
A	–	6	3	–	–	–
B	6	–	5	6	–	14
C	3	5	–	8	4	10
D	–	6	8	–	3	8
E	–	–	4	3	–	–
F	–	14	10	8	–	–

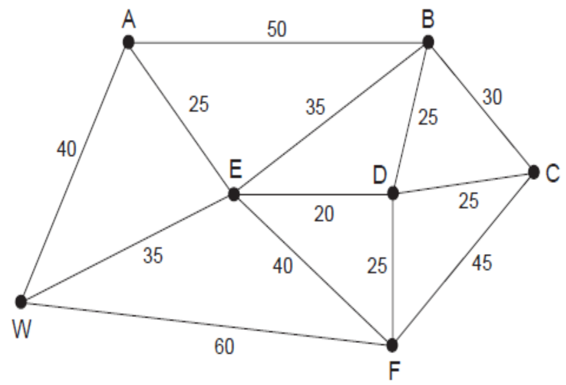
(i) On the table in the insert, use Prim's algorithm to find a minimum spanning tree. Start by crossing out row A. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

Question: How does the Prim's Algorithm differ when applied to a network?

Practice Questions:

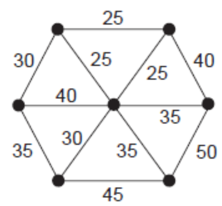
Entry Level Questions:

The owner of a caravan site has caravans positioned as shown in the diagram, with distances in metres between them, and wants to lay on a water supply to each of them. Use Kruskal's algorithm to determine how the caravans should be connected so that the total length of pipe required is a minimum.



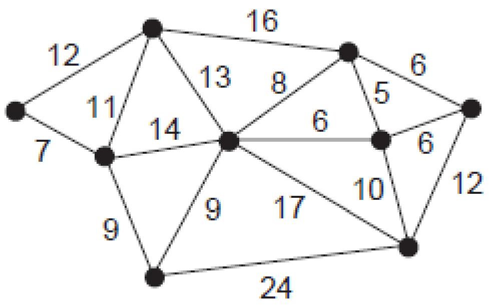
Answer: 155m

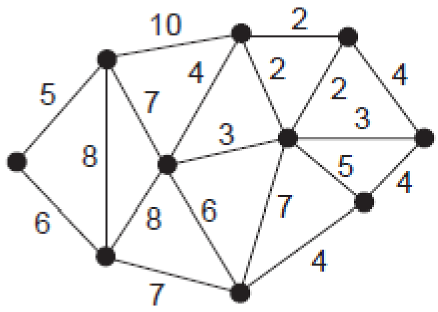
The warden of an outdoor studies centre wants to set up a public address system linking all the huts. The distances in metres between the huts are shown in the diagram opposite. How should the huts be linked to minimise the total distance?



Answer: 180m

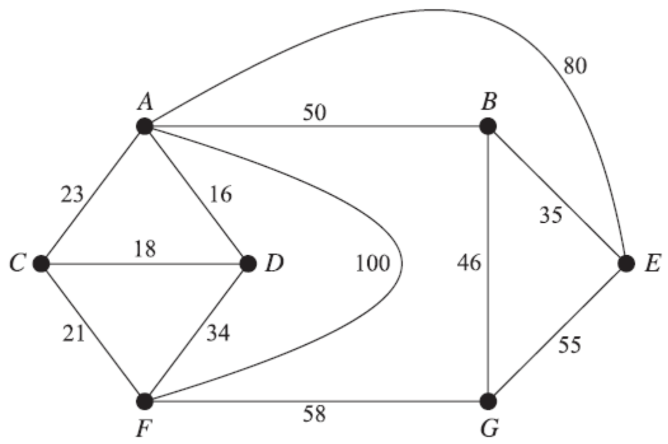
Use Prim's algorithm to solve the minimum connector problem for each of the graphs below.





Answer: 63 65

January 2008 – Question 3, parts i and ii on Kruskal's Algorithm:



(i) This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. [5]

(ii) Use your answer to part **(i)** to find the weight of a minimum spanning tree for the network with vertex G, and all the arcs joined to G, removed. Hence find a lower bound for the travelling salesperson problem on the original network. [3]

January 2007 – Question 4 part i – Matrix Version of Prim's Algorithm

The table shows the distances, in units of 100 m, between seven houses, A to G.

	A	B	C	D	E	F	G
A	0	4	5	3	2	5	6
B	4	0	1	2	4	7	6
C	5	1	0	3	4	6	7
D	3	2	3	0	2	6	4
E	2	4	4	2	0	6	6
F	5	7	6	6	6	0	10
G	6	6	7	4	6	10	0

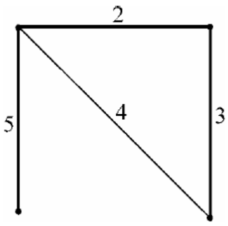
(i) Use Prim's algorithm on the table in the insert to find a minimum spanning tree. Start by crossing out row A. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

Specimen Paper – Question 2 – Kruskal's Algorithm

This question is about a simply connected network with at least three arcs joining 4 nodes. The weights on the arcs are all different and any direct paths always have a smaller weight than the total weight of any indirect paths between two vertices.

(i) Kruskal's algorithm is used to construct a minimum connector. Explain why the arcs with the smallest and second smallest weights will always be included in this minimum connector. [3]

(ii) Draw a diagram to show that the arc with the third smallest weight need not always be included in a minimum connector. [4]

2 **(i)** Using Kruskal's algorithm, the arc of least weight is chosen first and so is certainly included	B1		For identifying the first choice
The arc of second least weight is chosen next	B1		For identifying the second choice
since just two arcs cannot form a cycle	B1	**3**	For correct justification
(ii) 	B1		For any connected graph with 4 nodes and at least 3 arcs
	M1		For including a cycle
	A1		For a network having the required property
	A1	**4**	For making the minimum connector clear
		$\boxed{7}$	

Additional Questions:

Miscellaneous Exercise 3 from the Decision Mathematics 1 text book. Questions 1, 3 and 8.